Half-a-Century Evolution of Dynamic Game Theory
--A Personal Retrospective--

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Why Half-a-Century (from about 1970)?

Focus on Nonzero-sum Non-cooperative Differential/Dynamic Games

NOTE: A Personal Retrospective !!

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Why Half-a-Century (from about 1970)?

3 papers around 1970 kicked off the evolution:

A.W. Starr and Y.-C. Ho: Nonzero-sum differential games, JOTA, 3(3):184-206, 1969.

A.W. Starr and Y.-C. Ho: Further properties of nonzero-sum differential games, JOTA, 3(4):207-219, 1969.

M. Simaan and J.B. Cruz, Jr.: On the Stackelberg strategy in nonzero-sum games, JOTA, 11(5):533-555, 1973.

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---Quotes are from AI Overview---

Evolution of the field of differential/dynamic games

- "Since the 1970s, differential game theory has evolved from its initial focus on deterministic, military pursuit-evasion models into a sophisticated, robust multidisciplinary framework. It merges optimal control and strategic interaction, allowing analysts to model continuous-time, dynamic decision-making where agents' optimal strategies change over time based on the actions of all participants."
- "Dynamic game theory transitioned from a primarily theoretical construct to an applied discipline across economics, engineering, and biology. Since the 1970s, key advancements have deepened our understanding of how strategic interactions unfold over time, leading to significant mathematical modeling."

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Key dates in evolution (non-technical)

- 1972: *International Journal of Game Theory*, founded by Oskar Morgenstern.
- 1984: *International Symposia on Dynamic Games and Applications* (ISDGA), biennial since 1984 (skipping the COVID year 2020).
- 1989: *Games and Economic Behavior* launched
- 1990: Helsinki/Espoo meeting in June: Decision to formally establish the Society (*ISDG*), and to launch the *Annals* of the Society **From AI Overview:** "The **International Society of Dynamic Games (ISDG)** is a global non-profit professional organization founded in 1990 to advance the theory and application of dynamic games. A dynamic game is a competitive or cooperative decision-making process involving several agents in dynamic interaction over time, with applications in economics, biology, computer science, and engineering."

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- 1994: First volume of *Annals* published (T. Başar & A. Haurie, editors)
- 1999: Sister society—*Game Theory Society*—established
- 2008: International Conf Game Theory and Management launched (St Petersburg)

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From the memory lane: International Conference on Game Theory and Management, St. Petersburg, June 26-27, 2008

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- 2008: International Conf Game Theory and Management launched (St Petersburg)
- 2011: SV Book Series: Static & Dynamic Game Theory: Foundations and Applications
- 2011: *Dynamic Games and Applications* launched (G. Zaccour, EiC)
- 2018: *Handbook of Dynamic Game Theory, Vols I & II*, Springer, appeared (T. Başar & G. Zaccour, editors)

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Highlights (technical)

- Different solution concepts (Nash, Stackelberg, …)
- Informational non-uniqueness
- Robustness and sensitivity to model imprecision
- Incentivization / inducement of desired behavior
- Online and offline computations
- Potential games / Strategic equivalence to teams
- Handling asymmetric information within large populations—mean field approach
- Learning schemes for multi-agent systems
- What lies in the future

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Coming back to: Why Half-a-Century?

3 papers around 1970 kicked off the evolution; 2 on NE & 1 on SE:

A.W. Starr and Y.-C. Ho: Nonzero-sum differential games (NZS DGs), JOTA, 3(3):184-206, 1969.

------------: Further properties of NZS DGs, JOTA, 3(4):207-219, 1969.

M. Simaan and J.B. Cruz, Jr.: On the Stackelberg strategy in nonzero-sum games, JOTA, 11(5):533-555, 1973.

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The 1970s

Three publications, "claiming" LQNZSGs admit only linear (in state) NE policies:

A. Friedman: Differential Games, Chapter 8, Wiley, New York, 1971.

M.H. Foley and W.E. Schmitendorf: On a class of non-zero-sum linear-quadratic differential games, JOTA, 7(5):357-377, 1971.

D.L. Lukes: Equilibrium feedback control in linear games with quadratic costs, SIAM Journal on Control, 9(2):234-252, 1971.

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Uncountably many nonlinear NE for LQNZSDGS*

Perhaps the simplest DT LQNZSDG: scalar, 2-player, 2-stage, P_u acts twice, P_v once

$x_1 = x_0 + u_0 + v_0$; $x_2 = x_1 + u_1$; $u_0 = \gamma_0(x_0)$, $u_1 = \gamma_1(x_0, x_1)$, $v_0 = \gamma_2(x_0)$ [CL information]

$L^1(u, v) = (x_2)^2 + (u_1)^2 + (u_0)^2$ and $L^2(u, v) = (x_2)^2 + (v_0)^2 + \beta(u_1)^2$, $\beta \geq 0$ [quadratic losses]

Nash equilibrium: For $\beta \neq 1$, and given any $M > 0$ such that $|x_0| \leq M$, there exist uncountably many NE, linear and nonlinear, with one such set being (with $p((1-\beta) \neq 0$ and term under √ nn):

$\gamma_1^*(x_0, x_1) = -(1/2)x_1 + p\,(x_1)^2 - p\,[x_0 + \gamma_0^*(x_0) + \gamma_2^*(x_0)]^2$

$\gamma_0^*(x_0) = [1/12p(1-\beta)][[(3/4)(1+\beta) + 9/2] - \sqrt{\{[(3/4)(1+\beta) + 9/2]^2 + 36p(1-\beta)\, x_0\}}]$

$\gamma_2^*(x_0) = -x_0 - [1/4p\,(1-\beta)][[(3/4)(1+\beta) + 9/2] - \sqrt{\{[(3/4)(1+\beta) + 9/2]^2 + 36p(1-\beta)\, x_0\}}]$

provided that $0 < p(1-\beta) \leq [(1-\beta)/4 + 3/2]^2/4M$ **OR** $-[(1+\beta)/4 + 3/2]^2/4M \leq p(1-\beta) < 0$

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*T. Başar. A counter example in linear-quadratic games: Existence of non-linear Nash solutions. *Journal of Optimization Theory and Applications,* 14(4):425--430, Oct 1974.

13

Uncountably many nonlinear NE for LQNZSDGS*

Perhaps the simplest DT LQNZSDG: scalar, 2-player, 2-stage, P_u acts twice, P_v once

$x_1 = x_0 + u_0 + v_0$; $x_2 = x_1 + u_1$; $u_0 = \gamma_0(x_0)$, $u_1 = \gamma_1(x_0, x_1)$, $v_0 = \gamma_2(x_0)$ [CL information]

Corresponding NE costs (with β =0, for simplicity):

$L^1(u^*,v^*) = (49/16p^2)(1/8)\{-1 + \surd[1 + (64p/49)x_0]\}^2$
$L^2(u^*,v^*) = (1/16)\,(49/16p^2)\,\{-1 + \surd[1 + (64p/49)x_0]\}^2 + [(21/16p)\,\{-1 + \surd[1 + (64p/49)x_0]\} - x_0]^2$

Note: At high gain ($p \to \infty$), P_u drives his NE cost to zero, to the disadvantage of P_v.
And p=0 corresponds to strongly time-consistent (subgame perfect) NE.

$\gamma_2^*(x_0) = - x_0 - [1/4p\,(1- \beta)][[(3/4)(1+ \beta) + 9/2]-\surd\{[(3/4)(1+ \beta) + 9/2]^2+36p(1- \beta)\,x_0\}]$

provided that $0<p(1- \beta) \le [(1- \beta)/4 + 3/2]^2/4M$ OR $-[(1+ \beta)/4 + 3/2]^2/4M \le p(1- \beta) < 0$

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Note: At high gain ($p \to \infty$), P_u drives his NE cost to zero, to the disadvantage of P_v.
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Quote from JOTA'74:

"There will exist uncountably many linear as well as nonlinear Nash equilibria for multistage LQNZS games with the classical CL information structure for at least one of the players. This implies that equilibrium solutions of deterministic multi-person multi-objective decision problems will not be unique in general, which is a serious threat to the validity of the existing results in the literature on game theory."

*T. Başar. A counter example in linear-quadratic games: Existence of non-linear Nash solutions. *Journal of Optimization Theory and Applications,* 14(4):425--430, Oct 1974.

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How to remove "informational non-uniqueness"*,o

Bring imprecision to the model (state dynamics) through vanishing random perturbation with full support and independent across time. With N players, and nonlinear dynamics:

$x_{t+1} = f_t(x_t, u_t) + w_t^{\varepsilon}$, $t = 0, 1, \ldots$; $u = (u^1,..,u^N)$, $u_t^i = \gamma_t^i(x_{[0,t]})$, $i=1,..,N$, $\varepsilon > 0, \to 0$

$\{w_t^{\varepsilon}\}$ independent zero-mean sequence with +ve probability on every subset of state space

Loss function for player i (over t = 0, . . ., T): $L^i(x_{[0,T]}, u_{[0,T]}) = \sum_{t \in [0,T]} g_{i,t}(x_t, u_t)$

Take expectations with $u = \gamma(\cdot)$: $J_i(\gamma_i, \gamma_{-i})$ [normal form of game] → NE uses only x_t at t

Letting $\varepsilon \to 0$, we capture only one of the (uncountably many) NE of the deterministic DG

For the LQNZSDG, taking zero-mean Gaussian random sequence ➔ no dependence on ε.

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*T. Başar. On the uniqueness of the Nash solution in linear-quadratic differential games. *Int J Game Theory, 5(2/3):65-90, 1976 (submitted Aug 1974).*

o R. Selten. Reexamination of the perfectness concept for equilibrium points in extensive games. *Int J Game Theory, 4:25-55, 1975 (submitted Sept 1974).* ISDGA Plenary-TB 7/20/26

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How about mixed information structures across players?
E.g. N_1 players having CL information, and N_2 having OL information

Again full-support independent (across time) random perturbation eliminates informational non-uniqueness. For LQNZSDG this leads to unique NE, with N_2 players' NE policies linear in x_0, and N_1 players' linear in x_t and x_0.oo

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ooT. Başar. Nash strategies for M-person differential games with mixed information structures. *Automatica,* 11(5):547--551, Sept 1975.

Theory, 4:25-55, 1975 (submitted Sept 1974).

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How about the continuous-time case?*

Again, the simplest CT LQNZSDG: scalar, 2-player, unit interval, players enter additively:

$(d/dt)x_t = u_t + v_t$; $u_t = \gamma_u(t, x_{[0,t]})$, $v_t = \gamma_v(t, x_0)$ [CL information for P_u, OL for P_v]

$L^1(u, v) = (x_1)^2 + \int_{[0,1]} (u_t)^2\, dt$ and $L^2(u, v) = (x_1)^2 + \int_{[0,1]} (v_t)^2\, dt\ \beta \int_{[0,1]} (u_t)^2\, dt$, $\beta \geq 0$ [quadratic losses]

Nash equilibrium: Again, uncountably many NE, linear and nonlinear for P_u using memory, with one such set (linear in x_t and x_0) being (where p is a free parameter):*

$\gamma_u^*(t, x_t, x_0) = + p\, x_t - (1 - pt)\, v_t^*/2 - [1 + p(2 - t)]x_0/2$

$v^* = \gamma_v^*(x_0) = -(1 - 2/[2 + \beta + (1 - \beta)(e^p - 1)/p])\, x_0$

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*T. Başar. Informationally nonunique equilibrium solutions in differential games. *SIAM J Control and Optimization,* 15(4):636--660, July 1977 (submitted Feb 1975).

Removing informational non-uniqueness*

Perturb state dynamics with zero-mean full-support independent-increment process.

For the LQ, 2-player case:*

$dx_t = [F(t)\, x_t + G_1(t)\, u_t + G_2(t)\, v_t]\, dt + dw_t$, $t \geq 0$; $u_t = \gamma_u(t, x_{[0,t]})$, $v_t = \gamma_v(t, x_0)$

where $\{w_t\}$ is Brownian motion (or more generally a local martingale)

$L^1(u, v)$ and $L^2(u, v)$ are standard quadratic loss functions; with proper expectations taken we have the normal form $(J^1(\gamma_u, \gamma_v), J^2(\gamma_u, \gamma_v)) \rightarrow$ *Nash equilibrium (NE)*

Only one of the uncountably many NE of the deterministic DG constitutes (the unique) NE to this stochastic DG. This is neither strongly time consistent nor subgame perfect, since P_v has OL information.

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*T. Başar. Informationally nonunique equilibrium solutions in differential games. *SIAM J Control and Optimization,* 15(4):636--660, July 1977 (submitted Feb 1975).

DGs with high-order state dynamics

- Say 2 players (u,v), $x_{t+2} = f_t(x_{t+1}, x_t, u_t, v_t)$ (high-order state dynamics) $\{x_t\}$ n-dimensional
- Normally one would turn it into a 1st-order 2n-dimensional system $z_t := (x_t, y_t)$, $y_t = x_{t+1}$, and treat it as a standard NZSDG, which has informationally nonunique NE. To eliminate informational non-uniqueness, **only** the second sub-component of z dynamics is perturbed by zero-mean full-support, and not the full dynamics.
- For the resulting partially stochastic NZSDG, every NE is *informationally unique* and can be obtained through an iterative process that sweeps the time interval twice, in forward and retrograde time. And these policies entail full state memory, as opposed to those for games with 1st-order dynamics.*
- NOT implementable in infinite horizon, and also when there is a high population of players; here mean-field games framework comes into play – more on this later.**

*T. Başar "Informational uniqueness of closed-loop Nash equilibria for a class of nonstandard dynamic games," *JOTA*, 464(4):409-419, 1985

** T. Başar "Nash equilibria of networked games with delayed coupling in the high population regime," *Central European J Operations Research*, 33(2):415-427, June 2025.

Now coming to Stackelberg DGs

Paper that kicked off research in the topical area:

M. Simaan and J.B. Cruz, Jr.: On the Stackelberg strategy in nonzero-sum games, JOTA, 11(5):533-555, 1973.

- Real challenge to derive the (globally optimal) Stackelberg equilibrium when the leader has access to dynamic state information, at least using a direct approach that works for static NZSGs.
- Difference between (globally optimal) Stackelberg equilibrium and feedback SE (under CL information).

Informational non-uniqueness is a blessing for leader-follower (Stackelberg) DGs

- Given that deterministic NZSDGs with memory (equivalently, redundant information for a player) admit uncountably many NE, can one player in a leadership position (in the Stackelberg sense) use this to his advantage in attaining the lowest possible cost?
- A 2-player DT LQNZSDG with P1 ({u}) leader and P2 ({v_t}) follower, both having CL IS:

  $x_{t+1} = A(t)x_t + B_1(t)u_t + B_2(t)v_t$, t = 0,1,, N-1; $u_t = \gamma_1(t, x_{[0,t]})$, $v_t = \gamma_2(t, x_{[0,t]})$

  $L^1(u,v) = x_N{}^T Q_1(N)x_N + \sum_{[0,N-1]} x_t{}^T Q_1(t)x_t + u_t{}^T R_{11}(t)u_t + v_t{}^T R_{12}(t)v_t$

  $L^2(u,v) = x_N{}^T Q_2(N)x_N + \sum_{[0,N-1]} x_t{}^T Q_2(t)x_t + u_t{}^T R_{21}(t)u_t + v_t{}^T R_{22}(t)v_t$
- Well-acknowledged in the 1970s that obtaining the Stackelberg solution using direct approach was extremely challenging, if not impossible.
- Informational non-uniqueness provided a pathway for both deterministic NZSDGs as above and stochastic NZSDGs with redundant information (both DT and CT)

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Global Stackelberg equilibrium for DT LQNZSDG

- A 2-player DT LQNZSDG with P1 ({u}) leader and P2 ({v_t}) follower, both having CL IS:

  $x_{t+1} = A(t)x_t + B_1(t)u_t + B_2(t)v_t$, t = 0,1,, N-1; $u_t = \gamma_1(t, x_{[0,t]})$, $v_t = \gamma_2(t, x_{[0,t]})$

  $L^1(u,v) = x_N{}^T Q_1(N)x_N + \sum_{[0,N-1]} x_t{}^T Q_1(t)x_t + u_t{}^T R_{11}(t)u_t + v_t{}^T R_{12}(t)v_t$

  $L^2(u,v) = x_N{}^T Q_2(N)x_N + \sum_{[0,N-1]} x_t{}^T Q_2(t)x_t + u_t{}^T R_{21}(t)u_t + v_t{}^T R_{22}(t)v_t$
- It is sufficient for leader (L) to employ $u_t = \gamma_1(t, x_t, x_{t-1})$ to achieve the lowest cost.*
- If L has only partial state information, with redundancy, it's still possible to compute achievable performance using partial memory policies.**
- Global SE is in general nonunique, opening the way to build in robustness/sensitivity in design.∞

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*T. Başar and H. Selbuz. Closed-loop Stackelberg strategies with applications in the optimal control of multilevel systems. *IEEE Transactions on Automatic Control,* AC-24(2):166--179, April 1979.

** T. Başar. Performance bounds for hierarchical systems under partial dynamic information. *JOTA,* 39(1):67--87, Jan 1983.

∞ D. H. Cansever and T. Başar. Optimum/near optimum incentive policies for stochastic decision problems involving parametric uncertainty. *Automatica,* 21(5):575--584, Sept 1985.

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Global Stackelberg Equilibrium for CT LQNZSDG***

$dx/dt = A(t)x + B_1(t)u + B_2(t)v$, $x(0) – x_0$, $t \ge 0$; $u(t) = \gamma_1(t, x_{[0,t]})$, $v(t) = \gamma_2(t, x_{[0,t]})$

$L^i(u,v) = x(t_f)^T Q_{if} x(t_f) + \int_{[0,tf]} x(t)^T Q_i(t)x(t) + u(t)^T R_{i1}(t)u(t) + v(t)^T R_{i2}(t)v(t)]\, dt$; i = 1,2

- By employing a policy in the form:

  $\gamma_1(t, x(t), y(t), x) = P_1(t)x(t) + P_2(t)y(t) + P_3(t)x_0$, $dy/dt = C(t)y(t) + D(t)x(t)$, $y(0)=0$

  leader can force follower's L^2-optimizing response to jointly minimize $L^1(u,v)$.

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***T. Başar and G. J. Olsder. Team-optimal closed-loop Stackelberg strategies in hierarchical control problems. *Automatica*, 16(4):409-414, July 1980

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Global Stackelberg equilibrium for stochastic DGs

- 2-player single-act quadratic-Gaussian game where P1 (u) is leader with access to P2's action v and observation. *

  $u = \gamma_1(z_1, z_2; v)$, $v = \gamma_2(z_2)$; $z_i = Hx + w_i$, (x, w_1, w_2) independent zero-mean Gaussian vectors

  $L^i(u,v; x) = Q_i(u,v,x))$, i=1,2; Q_i are quadratic functions of (x,u,v), strictly convex in (u,v)
- Let $u^† = \gamma_1{}^†(z_1, z) = K_{11}z_1 + K_{12}z_2$, $v^† = \gamma_2{}^†(z_2) = K_2z_2$ jointly minimize (uniquely) leader's expected cost $E\{l^1(x,u,v)\}$.
- Then, there exists S such that with $u^* = \gamma_1{}^*(z_1, z_2; v) = \gamma_1{}^†(z_1, z) + S(v - K_2z_2)$, P2's best response is $v^* = \gamma_2{}^†(z_2) = K_2z_2$
- Hence the global SE (u^*,v^*) leads to overall optimum for the leader.*

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*T. Başar. Hierarchical decision making under uncertainty. In P. T. Liu, edt, Dynamic Optimiz & Math Economics, 205--221. Plenum Press, 1979

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- 2-player single-act quadratic-Gaussian game where P1 (u) is leader with access to P2's action v and observation. *

  $u = \gamma_1(z_1, z_2; v)$, $v = \gamma_2(z_2)$; $z_i = Hx + w_i$, (x, w_1, w_2) independent zero-mean Gaussian random vectors

  $L^i(u,v; x) = Q_i(u,v,x))$, $i=1,2$; Q_i are general quadratic functions of (x,u,v), strictly convex in (u,v)
- Let $u^t = \gamma_1^t(z_1, z) = K_{11}z_1 + K_{12}z_2$, $v^t = \gamma_2^t(z_2) = K_2z_2$ jointly minimize (uniquely) leader's expected cost $E\{l^1(x,u,v)\}$.
- Then, there exists S such that with $u^* = \gamma_1^*(z_1, z_2; v) = \gamma_1^t(z_1, z) + S(v - K_2z_2)$, P2's best response is $v^* = \gamma_2^t(z_2) = K_2z_2$
- Hence the global SE (u^*,v^*) leads to overall optimum for the leader.*
- Result extends to general Hilbert spaces (existence follows from *Separating Hyperplane Theorem*). **
- Also extends to multiple followers (mechanism design) and multiple levels of hierarchy.°°
- Also, dynamic stochastic systems where leader has access to one-step delayed information but no direct access to v.

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*T. Başar. Hierarchical decision making under uncertainty. In P. T. Liu, edt, Dynamic Optimiz & Math Economics, 205--221. Plenum Press, 1979

** T. Başar. Affine incentive schemes for stochastic systems with dynamic information. *SIAM J Control & Optimiz,* 22(2):199--210, March 1984

°° T. Başar. Stochastic incentive problems with partial dynamic information and multiple levels of hierarchy. *European J. Political Economy,* V:203--217, 1989

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Global Stackelberg equilibrium for stochastic DGs

- 2-player single-act quadratic-Gaussian game where P1 (u) is leader with access to P2's action v and observation. *

  $u = \gamma_1(z_1, z_2; v)$, $v = \gamma_2(z_2)$; $z_i = Hx + w_i$, (x, w_1, w_2) independent zero-mean Gaussian random vectors

"Hidden" Secret:

Redundancy of information enables leader to induce desired behavior on follower(s)

Ability to monitor follower(s)' action(s) and penalize if there is deviation from desired action(s)

- Result extends to general Hilbert spaces (existence follows from *Separating Hyperplane Theorem*). **
- Also extends to multiple followers (mechanism design) and multiple levels of hierarchy.°°
- Also, dynamic stochastic systems where leader has access to one-step delayed information but no direct access to v.

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°° T. Başar. Stochastic incentive problems with partial dynamic information and multiple levels of hierarchy. *European J. Political Economy,* V:203--217, 1989

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Global Stackelberg equilibrium for stochastic DGs

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Ability to monitor follower(s)' action(s) and penalize if there is deviation from desired action(s)

Another inducement scheme: Strategic Information Transmission

Leader (L) manipulates information of interest to follower (F) before transmitting it to him in a way the decision made by F (minimizing his own cost function) benefits L.
In simplest form: $z_2 = \gamma_1(z_1)$, F minimizes $E\{L^2(v, x)|z_2\}$

-M. Gentzkow and E. Kamenica. Bayesian persuasion. Amer. Econ. Rev., 101(6):2590-2615, 2011
-E. Akyol, C. Langbort, and T. Başar. Information-theoretic approach to strategic communication as a hierarchical game. *Proceedings of the IEEE, 105(2):205-218, February 2017*
-T. Başar. Inducement of desired behavior via soft policies. *Internat Game Theory Review*, 26(2):2440002, 2024
-NE: V. Crawford and J. Sobel. Strategic information transmission. Econometrica, 50:1431-1451, 1982

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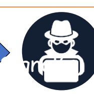
Another Inducement Scheme: Strategic Information Transmission

Information Flow
Through Signal(s)



Has Access to
Information of Interest

Seeks to Learn the
Information of Interest

Question: How to craft the information of interest to induce the receiving agent to perceive it as desired (by S) even when R is aware of the relationship between the received signal and the information of interest?

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Back to NE: Online computation

- **Policy iteration** (involves iteration based on individual best-response functions):[1,2,3,4]

  $\gamma_i^{t+1} \in Br_i(\gamma_{-i}^t)$, for each i, and t=0,1,…. respecting the network structure, including comm

  Fixed point of this multiple-valued map is NE

  Existence, uniqueness can be established under various structural assumptions[1,2,3,4]

  Convergence of various types of iterations (synchronous, asynchronous, sequential, randomness)

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[1]TB, "Decentralized multicriteria optimization of linear stochastic systems," TAC, April 1978.
[2]TB, "An eqm theory for multi-person DM with multiple probabilistic models," TAC, February1985.
[3]TB, S. Li, "Distributed algorithms for the computation of NE in linear SDGs," *SICON', May 1989*
[4]S.Yüksel, TB, Stochastic Teams, Games and Control under Information Constraints, Springer 2024

Online computation of NE

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  Fixed point of this multiple-valued map is NE

  Existence, uniqueness can be established under various structural assumptions[1,2,3,4]

- *Iteration in the action space* (involves expansion of individual sigma fields)

  Leading to possible asymptotic agreement, depending on richness of iteration[5]

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[2]TB, "An eqm theory for multi-person DM with multiple probabilistic models," TAC, Feb 1985.
[3]TB, S. Li, "Distributed algorithms for the computation of NE in linear SDGs," *SICON', May 1989*
[4]S.Yüksel, TB, Stochastic Teams, Games and Control under Information Constraints, Springer 2024
[5]S. Li, TB, "Asymptotic agreement and convergence of asynchronous algorithms," TAC, July 1987

Online computation of NE

- **Relaxation** (involves memory in the iteration based on individual best-response functions):[6]

  $\gamma_i^{t+1} \in Br_i(\gamma_{-i}^s$, s=t, t-1, .., t-$d_i$), for each i and t, where d_i is depth of memory for Player i

  Leads to improvement in convergence, such as faster rates.

- *Extremum Seeking* (sinusoidal excitation signals to estimate gradients and Hessians)

  Leading to convergence (asymptotic and finite-time) to NE with and w/o model info[7,8,9,10]

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[6]TB, "Relaxation techniques and asynchronous algorithms for on-line computation of noncooperative equilibria," *J Economic Dynamics and Control*, 11:531-549, Dec 1987.
[7]P.Frihaof, M. Krstic, TB, "Nash equilibrium seeking in noncooperative games," *TAC, May 2012.*
[8]T.R.Oliveira, V.H.P. Rodrigues, M.Krstic, TB, "Nash equilibrium seeking in quadratic noncooperative games under two delayed information-sharing schemes," *JOTA, 191:700-735, 2021*
[9]J.I..Poveda, M. Krstic, TB, "Fixed-time NE seeking in time-varying networks," *TAC, April 2023*
[10]T.R.Oliveira, M.Krstic, TB, "Extremum and NE seeking with delays and PDEs: Designs and Applications," *IEEE CSM, 39-87, April 2026*

Potential Games / Strategic Equivalence

- Subclass of NZSSGs (and possibly NZSDGs) allowing techniques of single objective optimization to compute NE.
- A 1996 paper by Monderer-Shapley[1] kicked off a new trend, with a large volume of follow-up work. Original idea goes back to Rosenthal.[2]

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[1] D. Monderer, L. Shapley. Potential games. Games and Economic Behavior, 14:124-143, 1996
[2] R.W. Rosenthal. A class of games possessing pure-strategy NE, *IJGT*, 2:65-67, 1973

Potential Games / Strategic Equivalence

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- A related, but more general, approach from 1974, applicable also to stochastic NZSDGs is establishment of strategic equivalence to teams through a transformation of objective fn of each player, independent of the policies of other players, making the objective functions of all players identical. If the resulting team has a unique pbp optimal team solution, also globally optimal, then unique NE.[3,4] Two examples are (stochastic) oligopoly and congestion games.

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[1] D. Monderer, L. Shapley. Potential games. Games and Economic Behavior, 14:124-143, 1996

[2] R.W. Rosenthal. A class of games possessing pure-strategy NE, *IJGT*, 2:65-67, 1973

[3] T. Başar. An extension of Radner's theorem to nonzero-sum games. *Proc. Allerton Conf on Circuit and System Theory*, 199-206, 1974

[4] T. Başar, Y.-C. Ho. Informational properties of the Nash solutions of two stochastic nonzero-sum games. *J Economic Theory*, 7(4):370-387, April 1974

Robustness (to model misspecification)

- Design of robust policies in DGs entail introduction of adversaries (whose capabilities are application dependent) working against original players within a zero-sum DG framework—turning an N-player game into a 2N-player one.[1,2]
- Equivalent to an N-player game where the loss functions of the players are of the risk-sensitive type (exponentiated stage-additive loss function).[4,5]
- One natural application area is network security[3]

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[1] T. Başar, P. Bernhard. H-infinity Optimal Control and Related Minimax Design Problems: A Dynamic Game Approach, Birkhäuser, 1992

[2] L.P. Hansen, T.J. Sargent. *Robustness,* Princeton University Press, 2008

[3] T. Alpcan, T. Başar. *Network Security: A Decision and Game Theoretic Approach.* Cambridge University Press, 2011.

[4] T. Başar. "Nash equilibria of risk-sensitive nonlinear stochastic differential games". *J. Optimization Theory and Applications,* 100(3):479-498, 1999.

[5] T. Başar. "Robust designs through risk sensitivity: An overview". *J. Systems Science and Complexity,* 34:1634-1665, Oct 2021.

Challenges in deriving NE and SE (model based)

- Almost complete theory for NE under CL perfect state (PS) information for all players (recursive computation in the spirit of DP when state dynamics are perturbed by diminishing noise), as well as under OL information (à la maximum principle). Also, an almost complete theory for SE when leader has OL information and followers either CLPS or OL. SE when leader has CLPS information is challenging through a direct approach, but an indirect approach with connections to mechanism design is feasible.[1,2]
- Other *dynamic (asymmetric)* information structures (s.a. local state, decentralized, measurement feedback): Extremely challenging! Possibly infinite-dimensional (even if, e.g., NE exists and is unique), even in linear-quadratic (LQ) NZS SDGs.[1]
- A few exceptions exist when asymmetric information can be decomposed into common and private information—game can then be "lifted" to one with only common information[3]

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[1] TB, G.J. Olsder. *Dynamic Noncooperative Game Theory,* SIAM, 1998 (1st edition Academic Press, 1982). .

[2] TB, "Affine incentive schemes for stochastic systems with dynamic information," *SIAM J Control & Optimiz,* 22(2):199-210, 1984.

[3] A. Gupta, A. Nayyar, C. Langbort, TB, "Common information based Markov Perfect Equilibria for linear Gaussian games with asymmetric information," *SIAM J Control & Optimiz,* 52(5):3228-3260, 2014.

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- There are also issues (computational and otherwise) in scaling up NE to a high population of players, even under symmetric information[4,5]

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[1] TB, G.J. Olsder, *Dynamic Noncooperative Game Theory,* SIAM, 1998 (1st edition Academic Press, 1982). .

[4] TB, R. Srikant, "A Stackelberg network game with a large number of followers," *JOTA,* 115(3):479-490, Dec 2002..

[5] E.Altman, TB, R. Srikant, "NE for combined flow control and routing in networks: Asymptotic behavior for a large number of users," *TAC,* 46(6):917-9300, June 2002.

Challenges in deriving NE and SE (model based)

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- Other *dynamic* information structures (s.a. local state, decentralized, measurement feedback): Extremely challenging! Possibly infinite-dimensional (even if, e.g., NE exists and is unique), even in linear-quadratic (LQ) NZS SDGs.
- Why? Strategic interaction!
- Each player (agent) has to *second guess* the information available to other players (and not to her) in her active neighborhood, as it could be useful to her in improving her performance. If all players are doing so, then this leads to an *infinite recursion* -- asymptotically *learning* relevant information through direct measurement of others' actions or through their impact on state or performance. Even obtaining *approximate* NE is a formidable task (such as placing restrictions on the dimensions of policies).

Any way to overcome/meet this challenge?

Mean-Field Games Approach[1,2,3] is the Answer

- Lift the N-player game up to an appropriate infinite-population one (assuming one exists, with possibly multiple populations, also respecting neighborhood relationships).
- Each generic agent faces a stochastic control problem, confronting a (sub)population which is exogenous to the agent, and not affected by the agent's actions.

****

[1]J.-M. Lasry, P.-L. Lions, "Mean field games," *Japan J. Math,* 2(1):229-260, 2007.

[2]M. Huang, P.E. Caines, R.P.. Malhamé, "Large population cost-coupled LQG problems with nonuniform agents: Individual-mass behavior and decentralized ϵ-Nash equilibria," *IEEE TAC,* 52(9):1560-1571, 2007.

[3]N. Saldi, TB, M. Raginsky, "Approximate Nash equilibria in partially observed stochastic games with mean-field interactions," *Mathematics of. Operations Research*, 44(3):1006-1033, 2019.

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Digression

- E.g., in local state dynamics $x_{i,(t+1)} = f_{it}(x_{it}, u_{it}, c_{it}(x_{-i,t}, u_{-i,t}), w_{i,t})$, $i = 1, \dots, N$,

  $c_{it}(x_{-i,t}, u_{-i,t}) = (1/|\mathcal{N}_{i,t}|) \sum_{j\in\mathcal{N}(i,t)} x_{j,t}$ where $|\mathcal{N}_{i,t}|$ is very large, even $\rightarrow \infty$
- *And/or in loss function* for player i (over t = 1, . . ., T) -- T could be ∞

  $L_i(x_{[1,T]}, u_{[1,T]}) = (1/T) \sum_{t \in [1,T]} g_{i,t}(x_{it}, u_{it}, k_{it}(x_{-i,t}, u_{-i,t}))$,

  $k_{it}(x_{-i,t}, u_{-i,t}) = (1/|\mathcal{N}_{i,t}|) \sum_{j\in\mathcal{N}(i,t)} x_{j,t}$ where $|\mathcal{N}_{i,t}|$ is very large, even $\rightarrow \infty$
- Here c_{it} and k_{it} are stochastic processes exogenous to agent (player) i
- Also, aggregate actions ($u_{j,t}$) of neighboring agents could enter the formulation

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Digression

- Such structures arise in many problems of interest that involve a large number of players, such as congestion control, sharing of common resources, and consensus formation[4,5]

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[4]TB, "A consensus problem in mean field setting with noisy measurements of target," Proc. 2018 ACC, pp. 6521-6526.

[5]H.Shen and TB, "Pricing under information asymmetry for a large population of users," Telecommunication Systems, 47(1-2):123-136, June 2011.

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Mean-Field Games Approach[1,2,3] is the Answer

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- Each generic agent faces a stochastic control problem, confronting a (sub)population which is exogenous to the agent, and not affected by the agent's actions.
- Generate the state process (and other possible aggregate quantities) under these optimal control policies and require consistency (entails solving a FP eq).
- Together with the optimal control policies, this leads to mean-field equilibrium (MFE).

****

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[3]N. Saldi, TB, M. Raginsky, "Approximate Nash equilibria in partially observed stochastic games with mean-field interactions," *Mathematics of Operations Research*, 44(3):1006-1033, 2019.

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- Together with the optimal control policies, this leads to mean-field equilibrium (MFE).
- It is possible to build in robustness through a risk-sensitive formulation (working with exponentiated loss functions for the agents)—connection to introducing an adversary agent, with generic agent now facing a zero-sum stochastic dynamic game.[4,5,6]

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[4]H. Tembine, Q. Zhu, TB, "Risk-sensitive mean field games," *IEEE TAC, 59*(4):835-850, April 2014.

[5]J. Moon, TB, "Linear-quadratic risk-sensitive and robust mean-field games," *IEEE TAC*, 62(3):1062-1077, March 2017; --"Risk-sensitive mean field games via the stochastic maximum principle," *Dynamic Games and Applications*, 9:1100-1125, 2019.

[6]N. Saldi, TB, M. Raginsky, "Approximate Markov-Nash equilibria for discrete-time risk-sensitive mean-field games," *Mathematics of Operations Research*, 45(4):1596-1620, Nov 2020.

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Mean-Field Games Approach is the Answer

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- Each generic agent faces a stochastic control problem, confronting a (sub)population which is exogenous to the agent, and not affected by the agent's actions.
- Generate the state process (and other possible aggregate quantities) under these optimal control policies and require consistency (entails solving a FP eq).
- Together with the optimal control policies, this leads to mean-field equilibrium (MFE)
- Finally, study the relationship between finite N and infinite N solutions—leading to ε-NE, thus resolving the formidable task of obtaining approximate NE for games with asymmetric information (such as local measurements only), where $\varepsilon \to 0$ as $N \to \infty$.

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Mean-Field Games with Stackelberg Leader[7]

- Stackelberg leader (L) faces an infinite-population of followers.
- For each policy of L, the followers play a mean-field Nash game as before, generating the Nash reaction set.
- L optimizes his expected cost on that reaction set, which in turn leads to followers' corresponding policies and the exogenous process at the followers' level.
- These, together with L's policy, lead to mean-field equilibrium (MFE).
- Study (as before) the relationship between finite (N) population of followers and infinite N solutions—with the policies in the MFE leading to (ε_S,ε_F) Stackelberg-NE, where ε_S provides the level of approximation to L's objective, and ε_F determines the level of approximation to each of N followers in the Nash game, where both $\varepsilon_S \to 0$ and $\varepsilon_F \to 0$ as $N \to \infty$.

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[7]J. Moon, TB, "Linear quadratic mean field Stackelberg differential games," Automatica, 97:200-213, Nov 2018.

Computational Aspects

- MFE-based approximate NE policy is scalable
- Computation of the mean field at NE requires solution of a fixed-point equation, which requires full modelling knowledge
- One way around this is for each agent to interact with a central coordinator (simulator) who collects state values and/or policies of the agents, computes the mean field, and broadcasts to all agents, who then update their policies based on the received MF, … and so on. With a finite number, L, of different populations of agents, L different MFs are computed.

Computational Aspects with Learning

- What if the agents do not know their own models? Then bring in RL for each agent into the iterative/learning process
- Parametrize the policies and optimize over the parameters, using e.g., policy gradient, respecting also computation and communication constraints[1,2,3]
- When explicit form of the agent's objective function is not available, its gradient can be computed only approximately, using e.g., **zero-order stochastic optimization (ZSO)**
- This will require further study of finite sample guarantees for the underlying algorithms
- It may be possible to avoid use of a central coordinator (simulator)[4]

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[1]T. Li, G. Peng, Q. Zhu, TB, "The confluence of networks, games, and learning: A game-theoretic framework for multiagent decision making over networks," *IEEE Control Systems Magazine, 42*(4):35-67, August 2022.

[2]T. Chen, K. Zhang, G.B. Giannakis, TB, "Communication-efficient policy gradient methods for distributed reinforcement learning," *IEEE TCNS*, 9(2):917-929, June 2022.

[3]B. Hu, K. Zhang, N. Li, M. Mesbahi, M. Fazel,TB, "Toward a theoretical foundation of policy optimization for learning control policies," *Annual Review of Control, Robotics, and Autonomous Systems*, 6:123-158, 2023.

[4]M.A. uz Zaman, S. Bhatt, A. Koppel, TB, "Oracle-free reinforcement learning in mean-field games along a single sample path," Proc. 25th Internat Conf AI & Statistics (AISTATS 2023), Valencia, Spain, April 25-27, 2023.

Computational Aspects with Learning

- What if the agents do not know their own models? Then bring in RL for each agent into the iterative/learning process

The scheme applies to MFGs with multiple populations (say M populations), where interaction of central coordinator is with a representative agent from each population, leading in the limit to M mean fields.[5]

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[5]M.A. uz Zaman, E. Miehling, T. Başar. "Reinforcement learning for non-stationary discrete-time linear-quadratic mean-field games in multiple populations." *Dynamic Games and Applications*, 13(1):118-164, March 2023

[2]T. Chen, K. Zhang, G.B. Giannakis, TB, "Communication-efficient policy gradient methods for distributed reinforcement learning," *IEEE TCNS*, 9(2):917-929, June 2022.

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Future of MFG research

MFG framework provides a versatile setting for addressing some complex decision-making problems in MASs.

A plethora of fruitful research opportunities exist toward broadening its applicability with also challenges on the theory front.[1,2]

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[1] T. Başar, B. Djehiche, and H. Tembine. *Mean-Field-Type Game Theory I: Foundations and New Directions.* Springer International Publishing AG, Switzerland, February 2026.

[2] T. Başar, B. Djehiche, and H. Tembine. *Mean-Field-Type Game Theory II: Applications.* Springer International Publishing AG, Switzerland, April 2026.

From Volume I on future of MFTG (theory)

Emerging directions in the MFTG theory continue to evolve, extending its reach into new and exciting directions. Some of the emerging areas and trends within the MFTG theory are:

- Distributionally Robust MFTGs
- MFTG Theory Driven by Mixed Noises
- Data-Driven MFTG Theory.
- MFTG Theory with Random Number of Interacting Decision-Makers
- MFTG Theory with Random Horizon Duration and Sleeping Mode
- MFTG Theory Driven by Non-Gaussian, Non-Poissonian, Non-Markovian Processes
- MFTG Theory with Changing Empathy Structures.
- MFTG Theory for Risk-Aware Audio-Enabled Machine Intelligence
- MFTG Theory for Strategic Machine Intelligence
  - Strategic Machine Intelligence for MFTG Theory:
- MFTG Theory-Based Risk Awareness:
- Variance-Aware MFTG Theory
- Quantile-Aware MFTG Theory
- Co-opetitive MFTG Theory
- Cooperative MFTG Theory
- Coalitional MFTG Theory

From Volume II on future of MFTG (appl)

- Agriculture with Manure Production
- Blockchain in Low-Connectivity and Low-Energy Areas
- Supply Chain Traceability
- Machine Intelligence in Internet-Deprived and Energy-Scarce Areas
- Healthcare Access
- Water Resource Management
- Psychological MFTGs
- Data-Driven MFTGs
- Distributionally Robust MFTGs
- Data-Driven Risk-Aware MFTGs
- Strategic Learning and Human Feedback
- Explainable Machine Intelligence (xMI)
- Climate Change Mitigation
- Human-Centric Urban Planning
- Personalized Treatments
- Cross-Disciplinary Collaborations
- Strategic Learning Integration
- Sustainable Technologies
- Healthcare and Epidemiology
- Ethical Considerations
- Real-Time Applications
- Education and Outreach
- Personalized Education
- Human-Machine Collaboration
- Regulatory Frameworks
- Quantum MFTGs

Far into the future for NZSDGs

What will the field of NZSDGs look like in 25 years (by 34th ISDGA) – AI Overview

- **In 25 years, the field of NZSDGs will shift from a highly theoretical, mathematically rigid discipline to the foundational operating system for autonomous interconnected global infrastructure, and to highly scalable multi-agent AI-driven stochastically complex paradigms.**
- As algorithms take over human decision-making, the field will move away from manual closed-form proofs of Nash equilibria and embrace real-time, path-dependent, and adaptive systems.

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What will the field of NZSDGs look like in 25 years (by 34th ISDGA) – AI Overview

Some details:

- Merging with Deep Multi-Agent Reinforcement Learning (MARL)
  - Model-free dynamic games: learning eqm strategies by interacting with changing environments
  - Universal approximators: deeply integrated, multi-scale network architectures
- Theoretical Shift: From Point Equilibria to "Dynamic Set Values"
  - Set-valued dynamic programming: capturing total set of possible outcomes
  - Path dependency: strategies deployed using the entire history
- Solution Architectures for Massive Multi-Agent Systems
  - Mean-field game scaling: flawless scaling approximating interactions among massive populations
  - Stochastic potential games: preventing chaotic systemic collapses
- Bounded Rationality and Machine-Driven Anomalies
  - Algorithm-to-Algorithm (A2A) dynamics: machines playing against machines
  - Paradoxical equilibria: uncovering highly counterintuitive non-human strategies

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What will the field of NZSDGs look like in 25 years (by 34th ISDGA) – AI Overview

Some details (continued):

- Generalization of MFGs
  - Heterogeneous (mega-)population models
  - Common noise integration
- Bridging Data-Driven Control and Game Theory
  - Data-to-game operators
  - Stochastic game topology: handling structural changes in the game itself
- Non-Smooth and Discontinuous Game Spaces
  - Viscosity solutions for multi-player systems with multiple equilibria
  - Hybrid differential-discrete games; quantum computing to speed up computation
- Dynamic Information Asymmetry
  - Computational delays and strategic limits of players
  - Epistemic DGs: what Player A (P-A) thinks what P-B knows about P-C's information/objectives

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Conclusion

Future is bright, with an expanding landscape, and with ample research opportunities, matching individual preferences !

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